SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2015

YEAR 12

ASSESSMENT TASK #3

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working Time – 120 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- In Questions 11-14, show all necessary working and/or calculations.
- Marks may not be awarded for untidy or badly arranged work.
- Answers should be in simplest exact form unless specified otherwise

Total Marks – 70

Section I (10 marks)

- Attempt Questions 1-10 on the multiple choice answer sheet provided.

Section II (60 marks)

- Attempt Questions 11-14
- Start a new booklet for each question.

Examiner: *A. Fuller*

Section I: 10 marks

Attempt Questions 1-10

Indicate which of the answers A, B, C, or D is the correct answer.

Use the multiple-choice answer sheet for Questions 1 – 10

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(1) Which of the following statements is INCORRECT?

(A) $\log a^n = n \log a$ (B) $\log ab = \log a + \log b$

(C) $\log(a-b) = \frac{\log a}{\log b}$ (D) $\log e = 1$

(2) Which of the following does $\frac{d}{dx}(e^4)$ equal?

(A) $4e^3$ (B) $\frac{1}{5}e^5$

(C) $4e^4$ (D) 0

(3) For what values of x is the curve $f(x) = 2x^3 + x^2$concave up?

(A) $x < -\frac{1}{6}$ (B) $x > -\frac{1}{6}$

(C) $x < -6$ (D) $x > -6$

(4) What is the period of $y = 4\sin(\frac{x}{3} + 2)$?

(A) 6π (B) $\frac{2\pi}{3}$

(C) 2π (D) 4

(5) If the Volume (V) is increasing at a decreasing rate:

(A) $\frac{dV}{dt} < 0,\ \frac{d^2V}{dt^2} < 0$ (B) $\frac{dV}{dt} > 0,\ \frac{d^2V}{dt^2} < 0$

(C) $\frac{dV}{dt} < 0,\ \frac{d^2V}{dt^2} > 0$ (D) $\frac{dV}{dt} > 0,\ \frac{d^2V}{dt^2} > 0$

(6) What is the greatest value of the function $y = 2 - 4\cos 2x$?

(A) 2 (B) 4

(C) 6 (D) 8

(7) Which of the following points would not form the vertices of a parallelogram with (1,1), (4,2) and (2,3)?

(A) (5,4) (B) (2,-1)

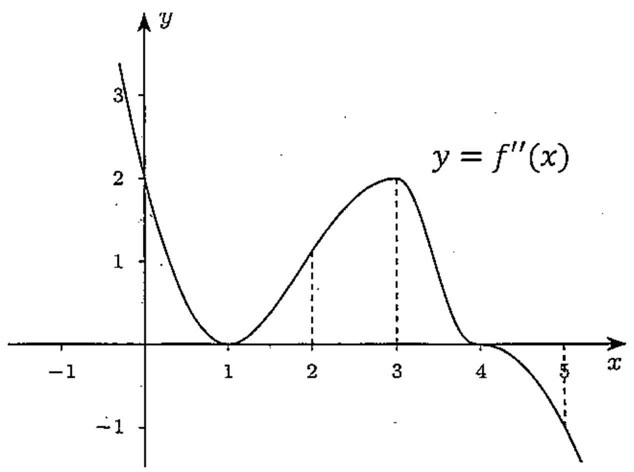
(C) (3,0) (D) (-1,2)

(8) An arc subtends an angle of 30 degrees at the centre of a circle with radius 30cm. The length of the arc is:

(A) 900 cm (B) $\frac{5\pi}{2}$ cm

(C) 450 cm (D) 5π cm

QUESTION 9 & 10 relate to the following sketch:



(9) The curve $y = f(x)$ has inflexion point(s) when:

(A) $x = 1\ ,\ 3$ (B) $x = 1\ ,\ 4$

(C) $x = 2$ (D) $x = 4$

(10) If $f'(1) = 0$ then $y = f(x)$ has:

(A) Horizontal point of inflexion when $x = 1$

(B) Maximum Turning point when $x = 1$

(C) Minimum Turning point when $x = 1$

(D) Discontinuity when $x = 1$

End of Section I

Section II: 60 marks

Attempt Questions 11-14

Question 11 (15 marks) Start a new booklet

(a) Differentiate the following with respect to x:

(i) $\tan 3x$ **1**

(ii) $x\cos x$ **2**

(iii) $\frac{1}{1+e^{2x}}$ **1**

(iv) $\ln\left(\frac{1+x^2}{e^x}\right)$ **2**

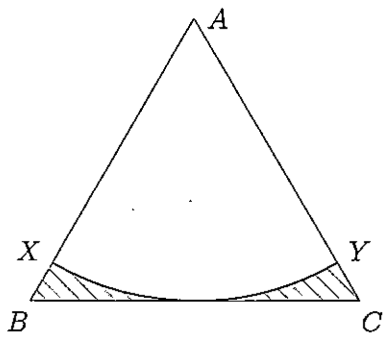
(b) When asked to calculate the volume of a solid generated when the area between the curve $y = f(x)$ and the x-axis is rotated about the x-axis between $x = 1$ and $x = 4$ a student correctly found that the volume was given by $V = \pi\int_1^4 4x\,dx$.

(i) Calculate this volume. **2**

(ii) If $f(x) > 0$ for $x > 0$, find the equation of the curve $y = f(x)$. **1**

(iii) Hence, find the area between the curve $y = f(x)$ and the x-axis between $x = 1$ and $x = 4$. **2**

(c) ΔABC is an equilateral triangle with sides of length 6 cm. An arc with centre A and BC as tangent cuts AB and AC at X and Y respectively.



(i) Show that the radius of the arc is $3\sqrt{3}$ cm. **2**

(ii) Find in exact form, the area of the shaded region. **2**

Question 12 (15 marks) Start a new booklet

(a) Find the following indefinite integrals:

(i) $\int \frac{1+e^{2x}}{e^{2x}} dx$ **2**

(ii) $\int \frac{e^{2x}}{1+e^{2x}} dx$ **2**

(iii) $\int \frac{1+e^{-2x}}{1+e^{2x}} dx$ **2**

(b) Use Simpson's rule with five function values to approximate $\int_1^5 \sin\frac{\pi}{x}\, dx$ to two decimal places. **3**

(c) An engine uses fuel at a rate of R litres per minute. The rate of fuel use t minutes after the engine starts operation is given by $R = 12 + \frac{10}{1+t}$.

(i) What is R when $t = 0$? **1**

(ii) What is R when $t = 9$? **1**

(iii) What value does R approach as t becomes large? **1**

(iv) Draw a sketch of R as a function of t. **1**

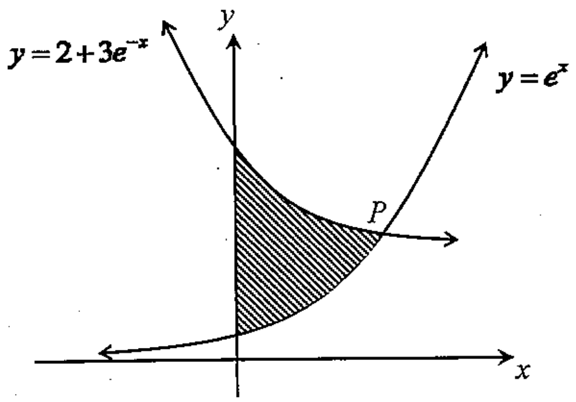
(v) Calculate the total amount of fuel burned during the first 9 minutes **2**

Question 13 (15 marks) Start a new booklet

(a) (i) Prove that $\frac{d}{dx}(\tan^3 x) = 3\sec^4 x - 3\sec^2 x$ **2**

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x\, dx$ **2**

(b) The diagram below shows the graphs of $y = e^x$ and $y = 2 + 3e^{-x}$ intersecting at the point P.



(i) Show that the curves intersect when $e^{2x} - 2e^x - 3 = 0$. **1**

(ii) Hence, show that the x-coordinate of the point P is $\ln 3$ **2**

(iii) Hence, find the exact area of the shaded region. **2**

(c) P and Q are the points on $y = \frac{1}{x}$ and $y = \frac{1}{\sqrt{x}}$ respectively where $x = 4$.

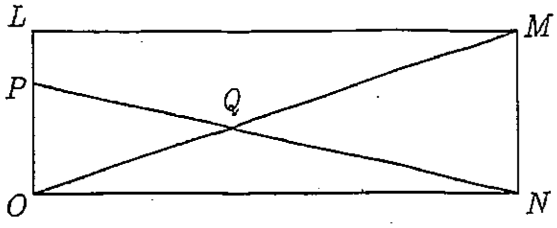
(i) Show that the tangents to their respective curves at P and Q are parallel. **2**

(ii) Show that the equation of the tangent at P is $x + 16y - 8 = 0$. **2**

(iii) Find the perpendicular distance between the tangents. **2**

Question 14 (15 marks) Start a new booklet

(a)



In the diagram $LMNO$ is a rectangle and $ON = 3(MN)$. The point P divides OL such that $OP = 2(PL)$.

(i) Show that ΔOQP is similar to ΔMQN. **1**

(ii) Show that $3(OM) = 5(QM)$. **2**

(iii) Show that $2(ON)^2 = 5(QM)^2$. **3**

(b) Consider the function $f(x) = \frac{x}{x^2+1}$.

(i) Show that $f(x)$ is odd. **1**

(ii) Find $\lim_{x\to\infty} f(x)$. **1**

(iii) Find any stationary points. **2**

(iv) Sketch $y = f(x)$ showing intercepts, asymptotes and stationary points. **1**

(v) Find the value(s) of k for which $f(x) = kx$ has three distinct solutions. **2**

(vi) Determine the number of real roots of the equation $x^2 - xe^{-x} + 1 = 0$ **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

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2015

HSC Task #3

Mathematics 2 Unit

Suggested Solutions & Markers' Comments

QUESTION	Marker
1 – 10	–
11	AYW
12	RE
13	AYW
14	JC

Multiple Choice Answers

1. C
2. D
3. B
4. A
5. B
6. C
7. B
8. D
9. D
10. C

Solutions for Multiple Choice Y12 2 Unit Task 3 2015

The mean score for this question was 6.45/10

Q1

$$\log(a-b) \neq \frac{\log a}{\log b}$$

C

A	0
B	6
C	53
D	27

Q2

$\log(e^4) = 0$ (as e^4 is constant)

D

A	6
B	1
C	25
D	54

Q3

$$f'(x) = 6x^2 + 2x$$

$$f''(x) = 12x + 2$$

$$12x + 2 > 0$$

$$\therefore x > -\frac{1}{6}$$

B

A	16
B	69
C	0
D	1

Q4

$$\text{Period} = \frac{2\pi}{\left(\frac{1}{3}\right)}$$

$$= 6\pi$$

A

A	71
B	9
C	5
D	1

Q5

$\frac{dV}{dt} > 0$; $\frac{d^2V}{dt^2} < 0$

(volume increasing) (rate decreasing)

B

A	0
B	75
C	4
D	6

Q6

$$-1 \leq \cos 2x \leq 1$$

$$-4 \leq -4\cos 2x \leq 4$$

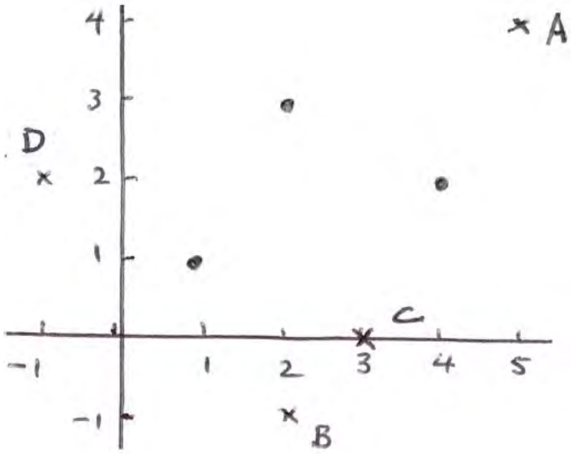
$$-2 \leq 2 - 4\cos 2x \leq 6$$

$\therefore$ Greatest value $= 6$

C

A	10
B	7
C	58
D	11

Q7



B

A	8
B	60
C	11
D	6

Q8

$$l = \frac{30}{360} \times 2\pi \times 30$$
$$= 5\pi \text{ cm}$$

D

A	1
B	3
C	0
D	82

Q9

Inflexion points can occur when $f''(x) = 0$ and $f''(x)$ changes sign on either side of the point.

$\therefore$ Point of inflexion at $x=4$.

D

A	6
B	49
C	8
D	22

Q10

10. $f'(1) = 0$

$\therefore$ Stationary point at $x=1$

As $f''(x) > 0$ on either side of $x=1$, the curve is concave up on either side of $x=1$

$\therefore$ Minimum turning point at $x=1$

C

A	50
B	3
C	24
D	9

Solutions and Marker's Comments: Question 11 (No $\frac{1}{2}$ marks given)

Question 11

a) (i) $\frac{d}{dx}(3\tan x) = 3\sec^2 3x$ 1 mark.

(ii) $\frac{d}{dx}(x\cos x) = u'v + v'u$

let $u = x$ $u' = 1$

let $v = \cos x$ $v' = -\sin x$

$= \cos x - x\sin x$

1 mark

1 mark

(iii) $\frac{d}{dx}\left(\frac{1}{1+e^{2x}}\right) = \frac{d}{dx}(1+e^{2x})^{-1}$

$= -(1+e^{2x})^{-2} \times 2e^{2x}$

$= \frac{-2e^{2x}}{(1+e^{2x})^2}$ 1 mark

(iv) $\ln\left(\frac{1+x^2}{e^x}\right) = \ln(1+x^2) - \ln e^x$

$= \ln(1+x^2) - x$

$\therefore \frac{d}{dx}\left(\ln\left(\frac{1+x^2}{e^x}\right)\right) = \frac{d}{dx}\left(\ln(1+x^2) - x\right)$ 1 mark

$= \frac{2x}{1+x^2} - 1$ 1 mark.

$= \frac{2x}{1+x^2} - \frac{(1+x^2)}{1+x^2}$

$= \frac{2x - 1 - x^2}{1+x^2}$

Comments:

- **Significant number of students lost 1 mark overall, as they did not write some sort of derivative notation for 2 or more subquestions in part a).**
- **Some students lost 1 mark in question a) ii) as they did not show any working of the product rule. Given that the particular question was 2 marks, students need to show working as well as state the final answer.**
- **Most students were able to get question a) i) and iii) correctly.**
- **Students who used the method in a) iv) above were more likely to get full marks than those who used the method of $\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$. This is because students who used the second method tend to make algebraic errors in their working.**
- **Significant number of students who wrote their final answer of 2 fractions into 1 in a) iv) made the careless error of not having a minus in front of x^2. Students lost 1 mark.**

b) i) $V = \pi \int_1^4 4x \, dx$

$= \pi \left[2x^2 \right]_1^4$ 1 mark

$= \pi \left(2(4)^2 - 2(1)^2 \right)$

$= \pi (32 - 2)$

$= 30\pi$ units^3 1 mark

(ii) $(f(x))^2 = 4x$

$f(x) = \pm\sqrt{4x} = \pm 2\sqrt{x}$

Since $f(x) > 0$ when $x > 0$

$\therefore f(x) = 2\sqrt{x}$ 1 mark

(iii) $2\int_1^4 x^{\frac{1}{2}} \, dx = 2\left[\frac{x^{3/2}}{3/2} \right]_1^4$ 1 mark

$= \frac{4}{3}\left[x^{3/2} \right]_1^4$

$= \frac{4}{3}\left(4^{3/2} - 1^{3/2} \right)$

$= \frac{4}{3}(8 - 1)$

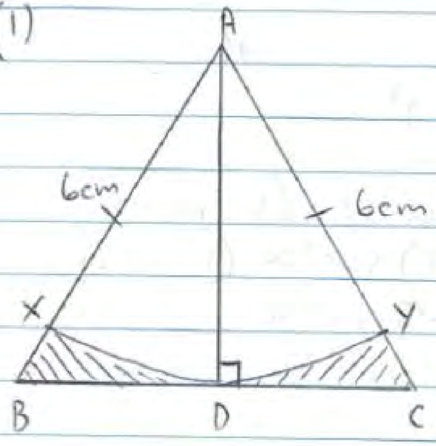
$= \frac{28}{3}$ units^2 1 mark

Comments:

- **Majority of the students did well in b) i) and ii).**
- **Significant number of students need to ensure that in b) ii), they don't forget to state the condition f(x) > 0 when x > 0. No marks was deducted for students who did not state that information.**
- **Many students lost marks in b) iii) especially if they integrated $(4x)^{\frac{1}{2}}$ or $\sqrt{4x}$ rather than $2\sqrt{x}$.**
- **A common careless mistake made by students in b) iii) was dividing by $\frac{3}{2}$ (see solution).**

  Students chose to multiply 2 by $\frac{3}{2}$ rather than multiply 2 by $\frac{2}{3} = \frac{4}{3}$.

(c) (i)



Construct AD such that $AD \perp BC$

* AD is the height of $\triangle ABC$
* AD is also the radius of the circle which point X and Y lies on
* DC is half of the side BC

$\therefore DC = 3cm$

$\therefore$ Using Pythagoras' theorem

1 mark

$$AD^2 = AC^2 - DC^2$$
$$= 6^2 - 3^2$$
$$= 27$$

1

$\therefore$ Radius $= AD = \sqrt{27}$ $\quad (AD > 0)$

$$= 3\sqrt{3}$$

$\therefore$ Radius $= 3\sqrt{3}$

(ii) Shaded Area $=$ Area$_{\triangle ABC}$ $-$ Area$_{\text{sector } AXY}$

$$= \frac{1}{2} \times AD \times BC - \frac{1}{2} r^2 \theta$$

1 mark

Since $\angle BAC$ is $60° = \frac{\pi}{3}$ radians

$\therefore$ Shaded Area $= \frac{1}{2} \times 3\sqrt{3} \times 6 - \frac{1}{2} \times (3\sqrt{3})^2 \times \frac{\pi}{3}$

$$= 9\sqrt{3} - \frac{9\pi}{2} \text{ cm}^2$$

1 mark

$$= \frac{18\sqrt{3} - 9\pi}{2} \text{ cm}^2$$

Comments

- **Significant number of students lost 1 mark as they did not state either AD is the radius of the sector or the words Pythagoras' theorem in c) i).**
- **Question C) ii) was poorly done by majority of the students. Mistakes were made for not substituting the correct numbers, not knowing how to find the shaded area or making careless errors in the arithmetic.**

12 (a)

(i) $\int \frac{1+e^{2x}}{e^{2x}} \cdot dx$

$= \int e^{-2x} + 1 \cdot dx$

$= -\frac{1}{2} e^{-2x} + x + c$ 2

(ii) $\int \frac{e^{2x}}{1+e^{2x}} \cdot dx$

$= \frac{1}{2} \int \frac{2e^{2x}}{1+e^{2x}} \cdot dx$

$= \frac{1}{2} \ln(1+e^{2x}) + c$ 2

(iii) $\int \frac{1+e^{-2x}}{1+e^{2x}} \, dx$

$= \int \frac{e^{2x}+1}{e^{2x}(1+e^{2x})} \cdot dx$

$= \int \frac{1}{e^{2x}} \cdot dx$

$= -\frac{1}{2} e^{-2x} + c$ 2

(b)

x	1	2	3	4	5
$f(x)$	0	1	0.866	0.707	0.587

$\int_1^5 \sin\left(\frac{\pi}{x}\right) \cdot dx$

$= \frac{3-1}{6}\left[0 + 4\times 1 + 0.866\right] + \frac{5-3}{6}\left[0.866 + 4\times 0.707 + 0.587\right]$

$= 3.049$ 3

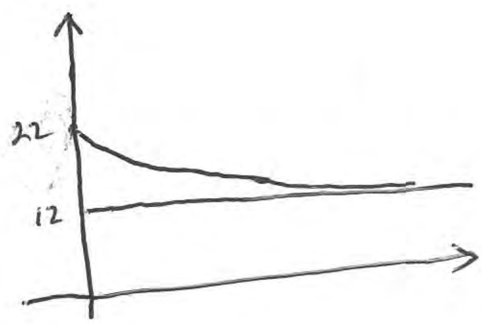
$= 3.05$

(c) (i) $R = 22$ 1

(ii) $R = 13$ 1

(iii) $R \to 12$ as $t \to \infty$ 1

(iv) 1



(v) Total $= \int_0^9 12 + \frac{10}{1+t} \cdot dt$ 2

$= \left[12t + 10\ln(1+t)\right]_0^9$

$= (108 + 10\ln 10) - [0 + 10\ln 1]$

$= 108 + 10\ln 10$

$= 131$ litres

COMMENTS Q12

12(a) (i) and (ii) well done

(iii) Most did not see that multiplying top and bottom by e^{2x} simplified the problem

(b) Most students made mistakes with the 'odds and evens' rule.

(c) Generally well answered but graphing was not well done.

Question 13 Solution and Marker's Comment: **(No $\frac{1}{2}$ marks are given)**

a) i) $\frac{d}{dx}(\tan^3 x) = \frac{d}{dx}(\tan x)^3$

$= 3\tan^2 x \sec^2 x$ (1) mark

$= 3(\sec^2 x - 1)\sec^2 x$ (1) mark.

Since $\tan^2 x + 1 = \sec^2 x$ $\quad = 3\sec^4 x - 3\sec^2 x$

$\therefore \tan^2 x = \sec^2 x - 1$

$\therefore$ LHS = RHS

(ii) From i) $\frac{d}{dx}(\tan^3 x) = 3\sec^4 x - 3\sec^2 x$

Rearranging the equation

$3\sec^4 x = \frac{d}{dx}(\tan^3 x) + 3\sec^2 x$

$\sec^4 x = \frac{1}{3}\left(\frac{d}{dx}(\tan^3 x) + 3\sec^2 x\right)$

$\therefore \int_0^{\frac{\pi}{4}} \sec^4 x \, dx = \frac{1}{3}\int_0^{\frac{\pi}{4}} \frac{d}{dx}(\tan^3 x) + 3\sec^2 x \, dx$ (1) mark

$= \frac{1}{3}\left[\tan^3 x + 3\tan x\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{3}\left[1 + 3 - 0 - 0\right]$

$= \frac{4}{3}$ (1) mark.

Comments:

- **Students who chose to differentiate** $\tan^3 x$ **a) i) by splitting into** $\tan^2 x \tan x$ **were not successfully able to show how it equalled to $3\sec^4 x$ - $3\sec^2 x$.**
- **Some students who had no idea how to differentiate $\tan^3 x$ in a) i) tried to "fudge" their working or just wrote the question down which led to no marks being awarded.**
- **Large number of students who did not use the result in part a) i) to work out what they should integrate, were not successful in determining the correct answer in part a) ii).**
- **In question a) ii), significant number of students did not multiply $\frac{1}{3}$ to 3 tanx or just multiplied $\frac{1}{3}$ to tanx (not 3tanx) which lead to an incorrect answer.**

(b) (i) Curves intersect when

$$2+3e^{-x} = e^{x}$$

$$e^{x} \times \left(e^{x}-3e^{-x}-2\right)=0 \times e^{x}$$ 1 mark.

$$e^{2x}-2e^{x}-3=0$$

(ii) $e^{2x}-2e^{x}-3=0$

$(e^{x}-3)(e^{x}+1)=0$

$\therefore e^{x}=3$ or -1 1 mark

Since $e^{x}>0$ for all values of x

$\therefore e^{x}=3$ 1 mark.

$\therefore x=\ln 3$

(iii) $\int_0^{\ln 3} (2+3e^{-x})-e^{x}\,dx$

$=\left[2x-3e^{-x}-e^{x}\right]_0^{\ln 3}$ 1 mark.

$=\left(2\ln 3-3\times\left(\tfrac{1}{3}\right)-3\right)-\left(0-3-1\right)$

$=2\ln 3-1-3+3+1$

$=2\ln 3$ units^2 1 mark

Comments:

- **Majority of the students did b) i) well.**
- **Students needed to show how ln 3 was derived in question b) ii) from the equation in b) i) rather than just substituting ln 3 into the equation which is proving. Marks was taken off, if students proved rather than show the answer.**
- **Students lost a mark if they did not state that e^{x} = -1 is not a valid solution in their result in b) ii). Students should state the e^{x} > 0 for all values of x. No extra marks was awarded or taken off for not stating the fact, but recommended for students to take note of.**
- **Significant number of students made careless errors in question b) iii) especially with e^{-x}. Students need to realise $e^{-x}=\frac{1}{e^{x}}$ and that when substituting ln 3, it becomes $\frac{1}{3}$ not -3. Other careless errors also included not subtracting (0 - 3 -1) which becomes +4, substituting 0 into $-3e^{-x}$ incorrectly or arithmetic errors.**

c) i) $y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$

$= -\frac{1}{x^2}$

At P where $x = 4$

$m_P = -\frac{1}{4^2} = -\frac{1}{16}$

$y = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-3/2}$

$= -\frac{1}{2\sqrt{x^3}}$

1 mark

At Q where $x = 4$

$m_Q = -\frac{1}{2\sqrt{4^3}} = -\frac{1}{2\times 8} = -\frac{1}{16}$

Since $m_P = m_Q$

$\therefore$ Tangents at P and Q are parallel. 1 mark

(ii) At point P, the gradient of tangent $= -\frac{1}{16}$ 1

$x = 4 \qquad y = \frac{1}{4}$

$\therefore$ Equation: $y - \frac{1}{4} = -\frac{1}{16}(x-4)$ 1

$16y - 4 = -x + 4$

$x + 16y - 8 = 0$.

(iii) $Q = (4, \frac{1}{2})$

Equation of tangent at P: $x + 16y - 8 = 0$.

$\therefore$ Perpendicular distance $= \left| \frac{x + 16y - 8}{\sqrt{1^2 + 16^2}} \right|$ 1 mark

$= \left| \frac{4 + 16(\frac{1}{2}) - 8}{\sqrt{1^2 + 16^2}} \right|$

$= \frac{4}{\sqrt{257}}$

$= \frac{4\sqrt{257}}{257}$ units 1 mark

Comments

- **Majority were able to find the gradients of the tangents at P and Q in c) i) correctly. However significant number of students lost marks due to not writing the notation m_p = derivative at P or m_q = derivative at Q; not stating "since $m_p = m_{q,}$ therefore tangents are parallel" or just working out the gradient of the tangents are equal but not stating the conclusion that "therefore the tangents are parallel."**
- **Question c) ii) was mostly done well by the students.**
- **Students lost marks in c) iii) for not knowing the perpendicular distance formula, substituting the wrong numbers into the formula or not rationalizing the denominator in the final answer.**

a) (i) In $\triangle OQP$ and $\triangle MQN$.

$\angle OQP = \angle MQN$ (vertically opp. $\angle s$)

$\angle POQ = \angle NMQ$ (alt. $\angle s$, $OP \parallel MN$) $\frac{1}{2}$

$\therefore \triangle OQP \sim \triangle MQN$ (equiangular) $\frac{1}{2}$ 1

(ii) $\frac{OQ}{QM} = \frac{OP}{MN}$ since $OL = MN$ and $OP = 2PL$ then $OP = \frac{2MN}{3}$

$\frac{OQ}{QM} = \frac{\frac{2}{3}MN}{MN}$

$\frac{OQ}{QM} = \frac{2}{3}$.

$OQ = \frac{2}{3}QM$ 1

Now, $3(OM)$

$= 3(OQ + QM)$

$= 3\left(\frac{2}{3}QM + QM\right)$

$= 3\left(\frac{5}{3}QM\right)$ 1

$= 5QM$

(iii) $ON^2 = OM^2 - MN^2$ (Pythagoras' Thm)

$= \frac{25}{9}QM^2 - \frac{ON^2}{9}$ since $3(OM) = 5(QM)$ and $ON = 3(MN)$

$ON^2 + \frac{ON^2}{9} = \frac{25}{9}QM^2$

$\frac{10(ON)^2}{9} = \frac{25}{9}(QM)^2$ 3

$10(ON)^2 = 25(QM)^2$

$\therefore 2(ON)^2 = 5(QM)^2$

Markers' Comments:

a) (i) Almost everyone received full marks in this question.

(ii) More than half of the students did not received full marks in this question. Many students were unable to see the ratios between sides.

(iii) Those students who achieved full marks in (ii) were able to ~~achieve~~ achieve full marks in this section.

Most students were unable to achieve full marks in this question.

b) $f(x) = \dfrac{x}{x^2+1}$

(i) $f(-x) = \dfrac{-x}{(-x)^2+1}$

$= -\dfrac{x}{x^2+1}$

Showing $f(-x) = \dfrac{-x}{(-x)^2+1}$
$= \dfrac{-x}{x^2+1}$ $\frac{1}{2}$

$-f(x) = -\dfrac{x}{x^2+1}$

Since $f(-x) = -f(x)$ $\therefore f(x)$ is odd 1

failed to show this will lose $\frac{1}{2}$ mark.

(ii) $\lim\limits_{x \to \infty} \dfrac{x}{x^2+1}$

$= 0$ 1

(iii) $f'(x) = \dfrac{x^2+1-2x^2}{(x^2+1)^2}$

$= \dfrac{1-x^2}{(x^2+1)^2}$ 1

Stat. points occur when $f'(x) = 0$

$\dfrac{1-x^2}{(x^2+1)^2} = 0$

$1-x^2 = 0$

$(1-x)(1+x) = 0$

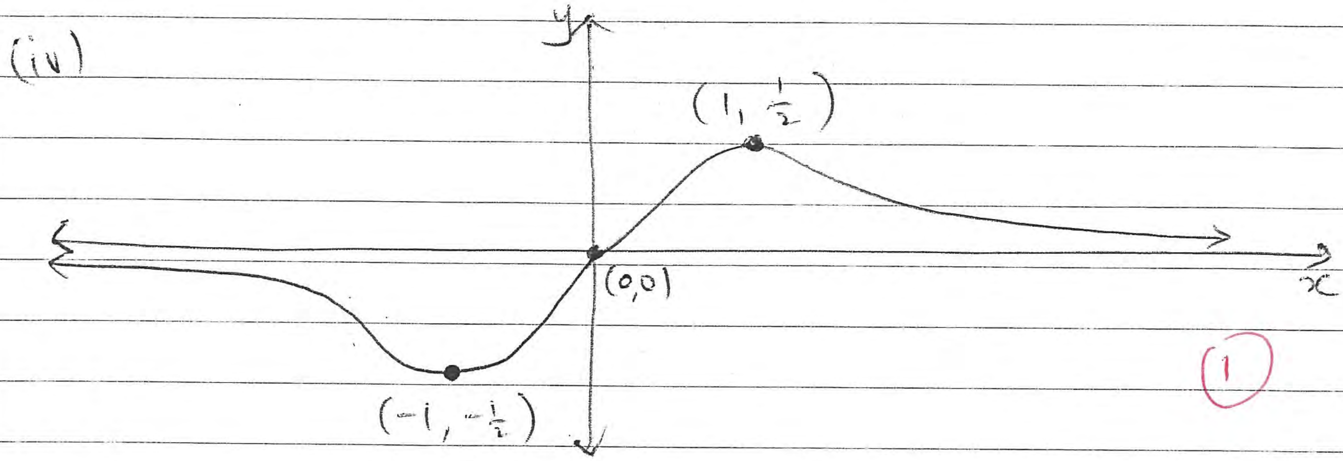
$x = \pm 1$

x	0.9	1	1.1
$f'(x)$	+	0	−

x	−1.1	−1	−0.9
$f'(x)$	−	0	+

$\therefore (1, \frac{1}{2})$ $\frac{1}{2}$ is a maximum point and 1
$(-1, -\frac{1}{2})$ $\frac{1}{2}$ is a minimum point.

(iv)



1

(v) $0 < k < 1$ since $f'(0) = 1$. 2

(vi) 0 real roots 1

$$x^2 - xe^{-x} + 1 = 0$$
$$x^2 + 1 = xe^{-x}$$
$$x^2 + 1 = \frac{x}{e^x}$$
$$e^x = \frac{x}{x^2+1}$$ 1

failed to show this will lose 1 mark

Markers' Comments:

b) (i) Almost all students were able to achieve full marks.
(ii) Almost all students were able to achieve full marks.
(iii) Almost all students were able to achieve full marks.
(iv) Almost all students were able to sketch the function.
(v) Most students were unable to find the range of k values.
(vi) Most students were unable to rearrange the equation $x^2 - xe^{-x} + 1 = 0$ into $e^x = \frac{x}{x^2+1}$ then use the sketch in (iv) to help solve this question.